News on the 2D wall impedance theory

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Context and motivation

- Beam-coupling impedances & wake fields (i.e. electromagnetic forces on a particle due to another passing particle) are a source of instabilities / heat load.

- In the LHC, low revolution frequency and low conductivity material used in collimators → classic thick wall formula (discussed e.g. in Chao’s book) for the impedance not valid e.g. at the first unstable betatron line (~ 8kHz):

  ⇒ need a general formalism with less assumptions on the material and frequency range to compute impedances (also for e.g. ceramic collimator, ferrite kickers).
Two dimensional models

Ideas:

- consider a longitudinally smooth element in the ring, of infinite length, with a point-like particle (source) travelling near its center, along its axis and with constant velocity $v$.

- integrate the EM force experienced by a test particle with the same velocity as the source, over a finite length.

$\Rightarrow$ Neglect thus all edge effects $\rightarrow$ get only resistive effects (or effects coming from permittivity & permeability of the structure) as opposed to geometric effects (from edges, tapering, etc.).

Main advantage: for simple geometries, EM fields obtained (semi-) analytically without any other assumptions (frequency, velocity, material properties – except linearity, isotropy and homogeneity).
Multilayer cylindrical chamber (Zotter formalism)

The chamber cross-section is shown with various components labeled, including:
- Ring-shaped source $\rho_m$ (inside: $\varepsilon_c^{(2)}$, $\mu^{(2)}$)
- Vacuum
- Pipe wall inner surface
- Cylindrical layers of different materials
- Beam

$\Rightarrow$ Source (in frequency domain, $k=\omega/v$) decomposed into azimuthal modes:

$$
\rho(r, \theta, s; \omega) = \frac{Q}{a \nu} \delta(r - a) \delta_p(\theta) e^{-jks} = \sum_{m=0}^{\infty} \rho_m = \sum_{m=0}^{\infty} \frac{Q \cos(m\theta) \delta(r - a_1) e^{-jks}}{\pi \nu a_1 (1 + \delta_{m0})}
$$
Multilayer cylindrical chamber: outline of Zotter formalism (CERN AB-2005-043)

- For each azimuthal mode we write Maxwell equation in each layer

\[ \text{div}\vec{D} = \rho_m, \]
\[ \text{curl}\vec{H} - j\omega\vec{D} = J_m, \]
\[ \text{curl}\vec{E} + j\omega\vec{B} = 0, \]
\[ \text{div}\vec{B} = 0, \]

with \( \vec{D} = \varepsilon_c\vec{E}, \quad \vec{B} = \mu\vec{H}, \)

where \( \varepsilon_c \) and \( \mu \) are general frequency dependent permittivity and permeability (including conductivity).

- We solve the resulting wave equations for the longitudinal components \( E_s \) and \( H_s \) using separation of variables, in cylindrical coordinates:

\[ E_s = \cos(m\theta)\left\{ e^{-jks} \right\} \]
\[ H_s = \sin(m\theta)\left\{ e^{-jks} \right\} \]

[linear combination of modified Bessel functions in \( r \)]

The transverse components are obtained from these, and there are 4 integration constants per layer.
Multilayer field matching: Matrix formalism

• Integration constants determined from field matching (continuity of the tangential field components) between adjacent layers. In the original Zotter formalism, one solves the full system of constants (4N equations)

⇒ Computationally heavy when more than two layers.

• Possible to relate the constants between adjacent layers with $4 \times 4$ matrix:

$$\text{Constants (layer } p+1) = M_p^{p+1} \cdot \text{constants (layer } p)$$

in the end

$$\text{Constants (last layer)} = M \cdot \text{constants (first layer)}.$$  

⇒ Only need to multiply $N - 1$ (relatively) simple $4 \times 4$ matrices and invert the final result, to get the constants.

Note: other similar matrix formalisms developed independently in H. Hahn, PRSTAB 13 (2010) and M. Ivanyan et al, PRSTAB 11 (2008).
Cylindrical chamber wall impedance

- Up to now we obtained the EM fields of one single azimuthal mode $m$.

- Sum all the modes to get the total fields due to the point-like source:

$$E_{s,tot}^{vac} = Ce^{-jks} \left[ K_0 \left( \frac{k}{\gamma} \sqrt{a_1^2 + r^2 - 2a_1 r \cos \theta} \right) - 2 \sum_{m=0}^{\infty} \frac{\alpha_{TM}(m) \cos(m\theta)}{1 + \delta_{m0}} I_m \left( \frac{ka_1}{\gamma} \right) I_m \left( \frac{kr}{\gamma} \right) \right]$$

$C$ and $\alpha_{TM}(m)$ are constants (still dependent on $\omega$).

- First term = direct space-charge $\rightarrow$ get the direct space-charge impedances for point-like particles (fully analytical).

- Infinite sum = “wall” part (due to the chamber). Reduces to its first two terms in the linear region where $ka_1 / \gamma << 1$ and $kr / \gamma << 1$. The “wall” impedances are then ($x_1$ = source coordinate, $x_2$ = test coordinate)

$$Z_{Wall}^l = \frac{jL \mu_0 \omega}{2\pi \beta^2 \gamma^2} \alpha_{TM}(0),$$

$$Z_{Wall}^x = \frac{jL Z_0 k^2}{4\pi \beta \gamma^4} [\alpha_{TM}(1)x_1 + \alpha_{TM}(0)x_2],$$

New quadrupolar term
Cylindrical chamber wall impedance results

- For 3 layers (10μm-copper coated round graphite collimator surrounded by stainless steel, at 450 GeV with b=2mm), dipolar and quadrupolar impedances (per unit length):

  $Z_{\text{dip}}^{\text{quad}}$ small except at very high frequencies.

  Importance of the wall impedance (= resistive-wall + indirect space-charge) at low frequencies, where perfect conductor part cancels out with magnetic images (F. Roncarolo et al, PRSTAB 2009).

- New quadrupolar impedance small except at very high frequencies.
Comparison with other formalisms

- In the single-layer and two-layer case, some comparisons done in E. Métral, B. Zotter and B. Salvant, PAC’07 and in E. Métral, PAC’05.

- For 3 layers (see previous slide), comparison with Burov-Lebedev formalism (EPAC’02, p. 1452) for the resistive-wall dipolar impedance (per unit length):

  Close agreement, except:
  - at very high frequency (expected from BL theory),
  - at very low frequency (need to be checked).
Multilayer flat chamber

\[
\begin{align*}
\varepsilon_c^{(p)}, \mu^{(p)} & \quad y = b^{(p)} \\
\varepsilon_c^{(1)}, \mu^{(1)} & \quad y = b^{(1)} \quad \Rightarrow \text{Chamber cross section (no a priori top-bottom symmetry)} \\
\varepsilon_c^{(2)}, \mu^{(2)} & \quad y = b^{(2)} \\
\varepsilon_c^{(0)}, \mu^{(0)} & \quad y = b^{(0)} \\
\text{Beam} & \\
\text{Plane } y = 0 & \\
\text{Vacuum} & \\
\text{Chamber materials} & \\
\varepsilon_c^{(-p)}, \mu^{(-p)} & \quad y = b^{(-p)} \\
\varepsilon_c^{(-2)}, \mu^{(-2)} & \quad y = b^{(-2)} \\
\varepsilon_c^{(-1)}, \mu^{(-1)} & \quad y = b^{(-1)}
\end{align*}
\]

\( \Rightarrow \) Source (in frequency domain) decomposed using an horizontal Fourier transform:

\[
\rho(x, y, s; \omega) = \frac{Q}{u} \delta(x) \delta(y - y_1) e^{-jks} = \frac{Q}{\pi u} \int_0^{+\infty} dk_x \tilde{\rho}(k_x, y, s; \omega) = \frac{Q}{\pi u} \int_0^{+\infty} dk_x \cos(k_x x) \delta(y - y_1) e^{-jks}
\]
Multilayer flat chamber: outline of the theory

- For each horizontal wave number $k_x$, solve Maxwell equations in a similar way as what was done in the cylindrical case, in cartesian coordinates (with source = $\tilde{\rho}$).

- Same kind of multilayer formalism (two 4x4 matrices in the end, one for the upper layers and one for the lower layers).

- Finally, instead of summing azimuthal modes, integrate over $k_x$.

After some algebra:

$$E_{s,tot}^{vac} = Ce^{-jks} \left[ K_0 \left( \frac{k}{\gamma} \sqrt{x^2 + (y - y_1)^2} \right) - 4 \sum_{m,n=0}^{+\infty} \frac{\alpha_{mn} \cos \left( n\theta - \frac{n\pi}{2} \right)}{(1 + \delta_{m0})(1 + \delta_{n0})} I_m \left( \frac{ky_1}{\gamma} \right) I_n \left( \frac{kr}{\gamma} \right) \right]$$

with $\alpha_{mn}$ given by (numerically computable) integrals over $k_x$ of frequency dependent quantities.
Flat chamber wall impedance

- Direct space-charge impedances are the same as in the cylindrical case (as expected).

- From wall part (infinite sums) → get wall impedance in linear region where $\frac{ky_1}{\gamma} \ll 1$ and $\frac{kr}{\gamma} \ll 1$ ($x_1$ & $y_1$ and $x_2$ & $y_2$ = positions of the source and test particles):

$$Z_{\|}^{\text{Wall},0} = \frac{jkZ_0L}{2\pi\beta\gamma^2} \alpha_{00}$$

$$Z_{x}^{\text{Wall},\text{dip}} = -\frac{jk^2Z_0L}{4\pi\beta\gamma^4} (\alpha_{00} - \alpha_{02}) ,$$

$$Z_{y}^{\text{Wall},\text{dip}} = \frac{jk^2Z_0L}{2\pi\beta\gamma^4} \alpha_{11} ,$$

$$Z_{x}^{\text{Wall,quad}} = \frac{jk^2Z_0L}{4\pi\beta\gamma^4} (\alpha_{00} - \alpha_{02}) ,$$

$$Z_{y}^{\text{Wall,quad}} = \frac{jk^2Z_0L}{4\pi\beta\gamma^4} (\alpha_{00} + \alpha_{02}) .$$

+ Constant term in vertical when no top-bottom symmetry:

$$Z_{y}^{\text{Wall},0} = \frac{jkZ_0L}{2\pi\beta\gamma^3} \alpha_{01}$$

Quadrupolar terms not exactly opposite to one another (≠ A. Burov – V. Danilov, PRL 1999, ultrarelativistic case)
Comparison to Tsutsui’s formalism

- For 3 layers (see parameters in previous figures), comparison with Tsutsui’s model (LHC project note 318) on a rectangular geometry, the two other sides being taken far enough apart:

\[ \Rightarrow \text{Very good agreement between the two approaches.} \]
Form factors between flat and cylindrical wall impedances

- Ratio of flat chamber impedances w.r.t longitudinal and transverse dipolar cylindrical ones → generalize Yokoya factors (Part. Acc., 1993, p. 511). In the case of a single-layer ceramic (hBN) at 450 GeV:

\[
\begin{array}{ccc}
10^4 & 10^6 & 10^8 \\
0.3 & 0.4 & 0.5 \\
0.6 & 0.7 & 0.8 \\
0.9 & 1.0 & 1.1 \\
\end{array}
\]

Frequency (Hz)

\[\text{Form factors} \]

- \(\text{Re}(F_{||})\), this theory
- \(F_{||}\), Yokoya (=1)
- \(\text{Re}(F_{\text{dip}}^x)\), this theory
- \(F_{\text{dip}}^x\), Yokoya (=\(\pi^2/24\))
- \(\text{Re}(F_{\text{dip}}^y)\), this theory
- \(F_{\text{dip}}^y\), Yokoya (=\(\pi^2/12\))

⇒ Obtain frequency dependent form factors quite ≠ from the Yokoya factors.
Conclusion

• For multilayer cylindrical chambers, Zotter formalism has been extended to all azimuthal modes, and its implementation improved thanks to the matrix formalism for the field matching.

  ⇒ The number of layers is no longer an issue.

• For multilayer flat chambers, a new theory similar to Zotter’s has been derived, giving also impedances without any assumptions on the materials conductivity, on the frequency or on the beam velocity (but don’t consider anomalous skin effect / magnetoresistance).

• Both these theories were benchmarked, but more is certainly to be done (e.g. vs. Piwinski and Burov-Lebedev, for flat chambers).

• New form factors between flat and cylindrical geometries were obtained, that can be quite different from Yokoya factors, as was first observed with other means by B. Salvant et al (IPAC’10, p. 2054).

• Other 2D geometries could be investigated as well.