Low Frequency Transverse Impedance Simulations of Collimators - Preliminary Results

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Introduction

- The low-frequency transverse impedance of the collimators constitutes a major part of the LHC impedance budget.
- The case of graphite collimators is not easy to assess with measurements and theoretical models have been evolving considerably over the last years.
- Conventional RF simulation tools face difficulties for frequency ranges below ~1 MHz, however there exist dedicated low frequency solvers, e.g. in CST EM Studio or Ansoft Maxwell. Currently we only have a license of the latter, which was therefore used. Typical applications of these tools are the design of AC transformers or the simulation of non-destructive testing devices using eddy currents.
The Model 1

- First a simple graphite structure with rotational symmetry was used: 5 mm half gap
- A two-wire simulation was performed. On all outside boundaries of the structure the magnetic field was set to be purely tangential (perfect conductor). The excitation is done not with a waveguide port as in RF simulations but by defining an ideal current source for each conductor
- In order to get the appropriate field pattern the two wires were excited in phase opposition
The Model 2

- To speed up simulations only the upper right quarter of the structure was modeled, with appropriate boundary conditions to make sure that we get the desired field symmetry.

- The maximum mesh size in the graphite was 2 mm, which corresponds to one skin depth at 1 MHz => upper limit of frequency range for graphite; for Cu with the same meshing one can go to about 10 to 100 kHz.

- The magnetic field is that of a dipole; it is concentrated in the plane of the two wires => related to horizontal transverse impedance.
Evaluation

- The code solves Maxwell's Equations directly without theoretical approximations as it seems.
- Once one knows the resulting current density the Ohmic losses can be calculated, which are proportional to the transverse impedance.
- In more detail: from the local current density and the resistivity the local Ohmic losses are calculated and integrated over the structure. Then the transmission $S_{21}$ is calculated, which gives the via the log formula an impedance, from which for a given wire spacing the transverse impedance is obtained.
The Physics Picture

- At DC all the current is flowing in the surrounding perfect conductor \(\Rightarrow\) the impedance is zero.

- Going from DC to low frequencies currents are induced in the less well conducting regions close to the beam due to Faraday's law \(\nabla \times E = -\frac{dB}{dt}\), \(E \sim f\), \(I \sim f\) \(\Rightarrow\) losses and thus impedance \(\sim f^2\). For the calculation of the transverse impedance one has to divide by \(f\) \(\Rightarrow\) \(Z_{TR} \sim f\) for low frequencies.

- At very high frequencies all the currents are flowing on a very thin layer on the inner conductor surface. The impedance increases with frequency with \(\sqrt{f}\) due to the skin effect \(\Rightarrow\) \(Z_{TR} \sim \frac{1}{\sqrt{f}}\) for high frequencies.

- Thus somewhere between low and high frequencies \(Z_{TR}\) must have a maximum; this maximum appears when the skin depth is about equal to the conductor thickness.
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- The situation of the currents leaving the surrounding perfect conductor and getting drawn to the beam at higher frequencies is illustrated below. Please note the log scale.
- At 10 kHz the graphite layer is roughly one skin depth thick; at 1 MHz the currents are concentrated in the innermost layers due to the skin effect. Not very clear here due to the scale...
Considered Structures

- Three geometries were considered
  - Rotationally symmetric structure for direct comparison with Burov-Lebedev formula
  - Two plates, as used in the collimator bench measurements
  - A simplified collimator cross-section

- Behind the structure there was either directly a perfect conductor or some space (30 to 220 mm) and a perfect conductor

- The conductor materials graphite (conductivity $6 \times 10^4$ S/m) and copper (conductivity $6 \times 10^7$ S/m) were used

- To limit the memory requirements 5 to 10 mm thick slices were modeled
Results – Comparison to Burov-Lebedev

- Very good agreement between simulation and the Burov-Lebedev theory for various structures with rotational symmetry.

- At 100 kHz and above the results for Cu become doubtful due to insufficient meshing in the copper.
Results - larger structure length

- Doubling the length from 5 to 10 mm did not noticeably affect the results.
- Good convergence was made sure of in the latter case (energy error < 1%).

![Graph showing impedance vs. frequency with data points for different structure lengths and simulations.](image)
Results – bench geometry 1

- Comparison between the ZTR expected for the bench measurements and Burov-Lebedev formula for a flat geometry (correction factor $\pi^2/8$ with respect to round geometry)

![Graph showing comparison between bench geometry measurements and Burov-Lebedev formula](image-url)

- Low frequency collimator two wire impedance simulation
  - Burov-lebedev formula for flat geometry and simulation of bench set-up
  - $r_w=0.5$ mm, $\Delta=3$ mm, half aperture=5 mm, coll thickness=10 mm, space=220 mm, $\gamma_{\text{graphite}}=664$ S/m

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Results – bench geometry 2

- The simulation as well as the analytical formula show an increase in ZTR when space is added between the graphite and the perfect conductor on the outside boundary. Going from a spacing of 30 to 220 mm does not have a large impact on ZTR.

![Graph showing the relation between frequency and real part of ZTR](image-url)

- Low frequency collimator two wire impedance simulation with various configurations:
  - Round graphite without space, analytical
  - Round graphite with 30 mm space, analytical
  - Round graphite with 220 mm space, analytical
  - Graphite block without space, col_bench_graphite_quarter2.mxwl
  - Graphite block with 30 mm space, col_bench_graphite_space_quarter2.mxwl
  - Graphite block with 220 mm space, col_bench_graphite_outerspace_quarter2.mxwl

Parameters:
- \( r_w = 0.5 \text{ mm}, \Delta = 3 \text{ mm}, \) half aperture = 5 mm, coll thickness = 10 mm, space = 30 mm, \( \gamma_{\text{graphite}} = 664 \text{ S/m} \)
Collimator cross-section

- For the simulation of the graphite collimators a quickly simplified geometry was used: The metallic support structure was modeled as a U-shaped channel structure. Graphite in grey, copper in red.
Collimator - Currents

- In collimator there are three regimes:
  - Low frequencies: skin depth large both in Cu and graphite, most of the current in the copper due to its smaller resistivity
  - Intermediate frequencies: skin depth in Cu comparable to Cu thickness => maximum impedance effect of Cu
  - High frequencies: graphite takes over currents => impedance

1 Hz: most of current and impedance in Cu

100 Hz: skin depth in Cu comparable to thickness

100 kHz: graphite takes over current
Results – collimator cross-section

- ZTR shows the characteristics of both an isolated graphite block and a copper block at a larger distance from the beam.
- ZTR is dominated by the graphite above 10 kHz and by the copper below a few 100 Hz.
- A three-layer analytical calculation shows a similar behaviour.
Summary

- The low-frequency solvers of commercial simulation packages can be used for evaluating the collimator transverse impedance at low frequencies.
- Very good agreement between the simulation and the Burov-Lebedev formula was obtained for structures with rotational symmetry.
- Preliminary results for a structure with graphite blocks as well as for a slice of an LHC graphite collimator were given.
- The latter showed characteristics of both the metallic support structure (low frequencies) and the graphite jaws (high frequencies).