Anomalous temperature dependence of the surface resistance of copper at 10 GHz


Indexing terms: Microwave components, Semiconductor devices and materials

Abstract: Effects of temperature on the surface resistance and volume resistivity of high-purity copper are investigated. The experimental investigation is based on accurate Q-factor measurements on a specially designed cavity resonator. Special experimental procedures were developed to yield the necessary accuracy. The results indicate an anomalous temperature variation of the surface resistance. The equivalent temperature coefficient of resistivity has a nonlinear temperature dependence.

1 Introduction
Discrepancies between the theoretical and experimental values of the surface resistance, with the latter considerably exceeding the former, were observed at high frequencies, particularly at millimetre wavelengths. The theoretical values were computed from the DC resistivities by using conventional skin-effect calculations. The discrepancies can be attributed to: surface roughness, surface preparation, anomaly of the skin effect and corrosion effects. They were investigated repeatedly [1-4].

It was also observed that the discrepancies and the anomalies of the skin effect are frequency-dependent [1-5]. The question therefore arises whether a discrepancy also exists between the theoretical and experimental temperature dependences of the surface resistance of copper at high frequencies. To find the answer, an investigation was made at 10 GHz in the temperature range 0-100°C. The theoretical temperature dependence was obtained by conventional skin-effect computation. The experiment showed that indeed such a discrepancy exists.

2 Surface resistance and DC resistivity
The surface resistance of conducting materials at high frequencies is the real part of the surface impedance. It is obtained by skin-effect computation from the volume resistivity of the material for DC. This resistivity is measurable for DC and very low frequencies only. It is not directly measurable for high frequencies owing to the skin effect. The surface resistance, in turn, can only be measured for high frequencies. The equations associated with the skin effect provide the link between the two quantities:

\[ R_s = \text{Re} \left[ Z_s \right] = \sqrt{\frac{\mu_0 \rho}{\pi f}} \quad (1) \]

where

- \( Z_s \) = surface impedance, \( \Omega \)
- \( R_s \) = surface resistance, \( \Omega \)
- \( \rho \) = volume resistivity for DC, \( \Omega \text{m} \)
- \( \mu \) = permeability for nonferromagnetic material, \( 4\pi \times 10^{-7} / \text{m} \)
- \( f \) = operating frequency, Hz

3 Measurement of the surface resistance
The experimental values of the surface resistance at high frequencies are most conveniently and accurately found, by measurement and evaluation of the Q-factors of cavity resonators. The Q-factors are directly related to the surface resistances of the cavity walls. It is imperative that the measurements are accurate, to make the small discrepancies between theoretical and experimental temperature variations observable. Two measures had to be taken to obtain the high accuracy: first, the use of a specially designed cavity resonator, and, secondly, the execution of carefully developed experimental procedures. Experience gained in previous similar experiments made the present investigation possible.

3.1 ‘Hi-cavity’ resonator
Rectangular-cavity resonators were found particularly suitable for determining surface resistances, because the wall surfaces are flat planes which can be conveniently polished. The design and the chosen dimensions represented a compromise to obtain a high Q-factor, to suppress higher-order modes, to suppress or reduce the effects of small gaps at the intersections of the walls and to minimise the effects of coupling elements located in the end walls.

The resonator is illustrated in Fig. 1a. For comparison, Fig. 1b shows the cross-section of the standard waveguide WG-16 (WR-90). The cavity resonator will be referred to as the ‘hi-cavity’, since \( b > a \), whereas WG-16 (WR-90) is called ‘lo-guide’, \( b < a \).

The Q-factor of the hi-cavity operating in the TE101 mode can be expressed in terms of the dimensions, as [7]

\[ Q_0 = \frac{\eta \pi}{4R_s} \left[ 1 + \left( \frac{a}{d} \right)^2 \right]^{3/2} \quad (2) \]

where \( \eta \) is the characteristic impedance of air dielectric (\( \eta = 120\pi \approx 377 \Omega \)) and \( R_s \) is the surface resistance of the walls. The equation suggests that the value of \( b \) should be large in
order to obtain a high $Q$-factor. Suppression of higher-order modes requires $a < b < \sqrt{3} a$. The value of $d$ should be large in order to minimise the effects of the coupling elements in the end walls. Hence, the width $a$ was chosen near to the cutoff value. The above design criteria minimise effects of the top and bottom walls on the $Q$. In such a resonator the large areas of the side walls, which can be uniformly polished, yield the major contributions to the total dissipated power.

### 3.2 Experimental approach and equivalent circuit

There are two possible approaches for finding $Q$-factors: one approach with the cavity in the transmission mode and the other based on reflection measurements. For the purpose of this experiment, the latter approach was chosen. This method is advantageous in the case of measuring the temperature effects when the cavity is immersed in water. Only one coupling and one waveguide are required.

In the reflection type operation, the $Q$-factor can be found from reflection coefficients measured in the input waveguide, as a function of frequency at and near resonance. It is advantageous to adjust the coupling (size of the window) so that the reflection coefficient is zero or very small at resonance.

A simplified analysis of the cavity system can be made by evaluating an equivalent circuit [6]. The basic structure and the simplified circuit are shown in Fig. 2. The cavity resonator is made of a section of 'high' waveguide with the characteristic impedance $Z_{O HI}$ connected by the coupling window to a standard WG-16 (WR-90) waveguide with $Z_{O LO}$. The equivalent circuit of the cavity is an RLC circuit connected by a lossless impedance transformer to the input transmission line with $Z_{O LO}$. At resonance, $R$ is transformed to $Z_{O LO}$. A resistor $r_t Z_{O LO}$ represents window losses on the waveguide side of the coupling window. Calculations indicate that, in this case, these losses are negligible small, and $r_t$ is disregarded.

The following sequence of equations shows the relationships used for the evaluation of the measurement results:

$$Z_{1,1} = R(1 + 2jQ \Delta f/f_0)$$

(3)

where

$$\Delta f = f - f_0$$

$$Z_{2,2} = Z_{1,1}/m^2$$

(4)

$$Z_{3,3} = r_t Z_{O LO} + Z_{2,2}$$

(5)

The characteristic impedances $Z_{O LO}$ and $Z_{O HI}$ are those of the two sections of waveguides. Setting $r_t = 0$ yields, for the input reflection coefficient $\Gamma_{in} = \Gamma_{3,3}$,

$$\Gamma_{in} = \frac{\Gamma_0 + jx(1 + \Gamma_0)}{1 + jx(1 + \Gamma_0)} = x = Q_0 \Delta f/f_0,$$

(6)

where $\Gamma_{in} = \Gamma_0$ for $\Delta f = 0$.

Eqn. 6 yields eqn. 7 for $Q$-factor

$$Q_0 = \frac{f_0}{|\Delta f|} \frac{1}{\sqrt{1 + |\Gamma|^2}}$$

(7)

The absolute values of the reflection coefficients were evaluated in the experiments. The pertinent equations are:

$$Q_0 = \frac{f_0}{|\Delta f|} \frac{1}{\sqrt{1 + |\Gamma|^2}}$$

(8a)

$$Q_0 = \frac{f_0}{|\Delta f|} \frac{1}{1 - |\Gamma|^2}$$

(8b)

$$Q_0 = \frac{f_0}{|\Delta f|} \frac{1}{1 - |\Gamma|^2}$$

(8c)

The use of one of these equations depends on the state of coupling associated with the size of the coupling window.

### 3.3 Circuit diagram and measurement procedures

Very systematic experimental procedures were adopted to give the required measurement accuracy. Instead of the customary determination of the $Q$-factors from the half-power frequencies, complete resonance curves were evaluated. Points near and far from resonance were not used in the evaluation, to keep errors low. The circuit diagram reflects the possibility of continuous checks of 'zero' and 'one' levels of the traces of the oscilloscope, which represent the corresponding absolute values of the reflection coefficients.

#### 3.3.1 Test setup:

The various elements of the test setup are evident in the circuit diagram shown in Fig. 3. A dual-trace oscilloscope (HP 1200B) was used as an indicator for finding the relationship between the absolute value of the reflection coefficient and the frequency. The precision attenuator (HP-X382A) allowed adjustment of the predetermined levels of the absolute values. A digital frequency counter (HP 5245M) allowed accurate measurement of frequencies. Several variable attenuators were used to adjust powers for optimising the power levels at the inputs of the crystal detectors $D_1$ and $D_2$. Their outputs were amplified in special low-noise preamplifiers $A_1$ and $A_2$.

#### 3.3.2 Experimental procedures:

A comparison method was used for finding the points of the resonance curves. The comparison was implemented by reducing the input power of detector $D_1$ to a specified level, by use of the precision attenuator, att. 1. This level corresponded to a specified absolute value of the input reflection coefficient. Subsequent frequency variation caused the absolute value of the input reflection coefficient and the corresponding signal entering $D_2$ to vary and to assume the same specified level as above, making the two traces of the oscilloscope coincide.
comparison approach, the beam deflections of the oscilloscope
and the powers entering the detectors were kept constant,
minimising measurement errors. The calibration of the
precision attenuator att. 1 was checked by bolometer, prior to
the experiment. A switch in front of detector D2 allowed
adjustments for 'zero' magnitude of the reflection coefficient.
Substitution of a short for the cavity, and setting att. 1 to
0 dB, allowed adjustment of the trace positions for magnitude
'1' of the reflection coefficient. The frequency variations were
introduced by adjusting the reflector voltage of the klystron
by a precision potentiometer. Written steps of the measure-
ment procedures were carefully followed to achieve uniform
timing of the various steps. Different sets of measurements
with increasing and decreasing temperature were made, and
the Q-values of the two sets were averaged. Effects of
temperature differences between inside and outside of the
cavity walls were practically eliminated.

4 Temperature dependence of surface resistance and
Q-factor
A temperature change of a cavity resonator causes linear
thermal expansion of the cavity dimensions. The effect can be expressed as

\[ l(T) = l(20)(1 + \gamma(T - 20)) \]  

(9)

where \( l(20) \) represents any cavity dimension at 20°C. The
cavity temperature is \( T \), and \( \gamma \) is the linear-thermal-expansion
coefficient. The latter is 16.5 \times 10^{-6} \text{ for copper [8].}

A change of the thermal expansion, in turn, causes a change of the resonant frequency of the cavity. The resonant frequency becomes

\[ f_0(T) = \frac{f_0(20)}{1 + \gamma(T - 20)} \]  

(10)
since

\[ f_0 = \frac{c}{2a} \sqrt{1 + \left(\frac{a}{d}\right)^2} \]

where

\[ \rho(T) = \rho(20)[1 + \alpha(T - 20)] \]  

(11)

where \( \rho(20) \) is its value at 20°C and \( \alpha \) is the temperature
coefficient of resistivity. The value of \( \rho(20) \) for the copper of
the cavity is known from DC measurements [8]. It is
1.7241 \times 10^{-8} \Omega m. For the temperature coefficient the
published value 0.00393 is used [9].

The thermal effect on the surface resistance is a combi-
nation of the thermal expansion and the change of the volume
resistivity. Substitution of eqns. 10 and 11 into eqn. 1 gives

\[ R_s(T) = R_s(20) \sqrt{1 + \alpha(T - 20)} \]  

(12)

The reference value of the surface resistance at 20°C is

\[ R_s(20) = \sqrt{\frac{l}{2\rho(20)}(l)} \]  

(13)

where \( f_0(20) \) represents any cavity dimension at 20°C. The
resonant frequency becomes

\[ f_0(T) = \frac{f_0(20)}{1 + \gamma(T - 20)} \]  

(14)

The preceding equations were used in the evaluation of the
experimental results.

5 Evaluation of Q-factors and surface resistance
Preparatory experiments using a slotted line for adjusting

![Circuitry for finding the temperature dependence of Qo](image)

Table 1: Measured values of Q-factor, resonant frequency, surface resistance and volume resistivity

<table>
<thead>
<tr>
<th>Temperature</th>
<th>Qol(T)</th>
<th>f_o</th>
<th>R_s(T)</th>
<th>p(T)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>8814</td>
<td>9.9386</td>
<td>0.02688</td>
<td>1.8408</td>
</tr>
<tr>
<td>20*</td>
<td>8667</td>
<td>9.9371</td>
<td>0.02733</td>
<td>1.9041</td>
</tr>
<tr>
<td>30</td>
<td>8525</td>
<td>9.9356</td>
<td>0.02779</td>
<td>1.9684</td>
</tr>
<tr>
<td>50</td>
<td>8254</td>
<td>9.9326</td>
<td>0.02870</td>
<td>2.1004</td>
</tr>
<tr>
<td>70</td>
<td>7999</td>
<td>9.9296</td>
<td>0.02961</td>
<td>2.2371</td>
</tr>
<tr>
<td>90</td>
<td>7756</td>
<td>9.9264</td>
<td>0.03054</td>
<td>2.3803</td>
</tr>
</tbody>
</table>

*Interpolated values

Further evaluation of the surface resistance yields the volume resistivity for the frequency of the experiment. At this high frequency the volume resistivity loses some of its significance because it is not directly measurable and, to be exact, it may not be uniform within the thin-skin layer. However, if it is compared with the volume resistivity at DC and low frequencies, its evaluation is useful and instructive as it clearly shows the effects of temperature.

For each temperature value (10, 30, 50, 70 and 90°C) ten measurements of the frequency for the absolute values of the reflection coefficients (−4, −6, −8, −10 and −18 dB) were made. The values of frequency for each pair were symmetrical on both sides of resonance. The frequencies for 0.2 < |f| < 0.75 were then evaluated. The resulting linearly averaged unloaded Q-factors, found by eqn. 8, are tabulated in Table 1 and plotted in Fig. 4. For comparison, the diagram includes the theoretical Q-factors found by eqn. 14 from the DC value of ρ(20) at the resonant frequency.

The values of the theoretical and the experimental surface resistances are also shown in the Figure. The data indicate that the experimental surface resistance at 20°C exceeds the theoretical value by about 5.1%. Discrepancies of this order of magnitude were previously observed. They are primarily caused by the combined effects of surface roughness, surface preparation and by a room temperature anomaly of the skin effect.

6 Temperature effects

Further evaluation of the surface resistance yields the volume resistivity for the frequency of the experiment. At this high frequency the volume resistivity loses some of its significance because it is not directly measurable and, to be exact, it may not be uniform within the thin-skin layer. However, if it is compared with the volume resistivity at DC and low frequencies, its evaluation is useful and instructive as it clearly shows the effects of temperature.

The above equation was derived from eqn. 14. The variation of the experimental temperature coefficient of resistivity does not vary linearly as a function of temperature, as is the case for DC. Fig. 6 shows the volume resistivity ρHF derived from the experimental Q-factor by

\[ a_{HF}(T) = \frac{[Q_0(20)]^2}{Q_0(T)} - 1 \].

This compares with

\[ \rho_{DC}(T) = \rho_{DC}(20)[1 + \alpha_{DC}(T - 20)] \].

The actual form of \( \rho_{HF}(T) \) can be approximated by a combination of linear and quadratic contributions, according to

\[ \rho_{HF}(T) = \rho_{HF}(20)[1 + \alpha_1(T - 20) + \alpha_2(T - 20)^2] \].

where \( \alpha_1 = 3.322 \times 10^{-3} \) and \( \alpha_2 = 5.376 \times 10^{-6} \).
for DC and low frequencies also shown in the Figure. The total discrepancy between $p_{HF}$ and $p_{DC}$ consists of two parts. One is a difference between $\alpha_{1}$ and $\alpha_{DC}$, which can be assumed to result from causes similar to those which are the origin of the discrepancy between the theoretical and experimental values of $R_{s}$. The second part is the nonlinear contribution associated with $\alpha_{2}$. This second part indicates that the conduction and field-penetration phenomena are different from those described in the lower-frequency region, by customary conduction and skin-effect theory.

![Graph showing $p_{HF}$ and $p_{DC}$]

Fig. 6 Volume resistivities

7 Conclusions

Experiments and their results have been described which show that discrepancies exist between the theoretical and experimental temperature dependences of the surface resistance of copper at 10 GHz. As a consequence, the volume resistivity, which is derived from the experimental surface resistance, also differs correspondingly from the DC resistivity at this frequency. While the temperature dependence of the volume resistivity at DC can be represented by a straight line and a constant temperature coefficient, the equation for the volume resistivity at high frequencies contains a quadratic term and the temperature coefficient varies as a function of temperature. Owing to this quadratic term, the HF resistivity exceeds the straight-line value at 100°C by about 3.1%. The deviation indicates differences between the basic conduction and field penetration phenomena at low and at extremely high frequencies.

8 References

5 DINGLE, R.B.: ‘The anomalous skin effect and the reflectivity of metals‘, Physica, 1953, 19, pp. 311–347
7 RAMO, S.: ‘Fields and waves in communication electronics‘ (John Wiley & Sons, 1957)

Book review

Aperture antennas and diffraction theory
E.V. Jull
Peter Peregrinus Ltd., 1981, 173 pp., £23.75
ISBN: 0–906048–52–4

This book, which is the outcome of a series of lectures given by the author over a number of years, falls naturally into two parts. The first part (Chapters 2–6) presents the well-established methods of analysing aperture antennas; namely, the use of Fourier transforms, Fresnel transforms, and Kirchhoff diffraction theory. The second part (Chapters 7–9) introduces the geometrical theory of diffraction as a means of solving aperture antenna problems. Two chapters are devoted to a number of applications and this, together with a well written text, should make the book appealing to students in need of an introduction to this important area of antenna analysis.

As for the practising engineer or research scientist, the book has to be criticised for its lack of depth and the fact that much of the material is readily available elsewhere. In the first part, a chapter on near-field radiation effects includes a section on deriving near-field patterns from far-field patterns. This subject is dealt with in one page and completely overlooks the severe difficulties which can be encountered when attempting to construct the near field from a given far field which, in the vast majority of cases, will be known only approximately. It would have been better not to have introduced this topic at all, unless it could have been given more adequate treatment. Again, in the same Chapter, the effect of the directivity of a measuring antenna on the observed radiation pattern is discussed. This is a topic of keen interest to many experimentalists. However, the treatment is again too perfunctory to be of much use to the practising engineer.

The same criticism applies in the second part. Over the last few years a bewildering array of competing asymptotic diffraction theory techniques, applicable to aperture antenna analysis, have been established. Complete, as is the current propensity, with their abbreviations, namely, PO, GO, PTD, GTD, UTD, UAT, STD etc, they pose a formidable task for the novice to unravel. Prof. Jull does make an attempt to sort out this mess, but unfortunately does not go far enough to be of real benefit.

Some other topics I would have liked to have seen mentioned include the comparative crosspolar performance as predicted for circular and rectangular apertures, the combination of the geometrical theory of diffraction with integration techniques, coupling between apertures, irregular apertures and apertures in nonplanar surfaces.

This book is of modest size and it is a pity that the author did not extend his scope beyond a university course to make the book more appealing to a wider audience.

G.L. JAMES